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The polarization of waves reflected from the ionosphere

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RESEARCH DEPARTMENT

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D. Maurice

Head of Research Department

P. Knight, B.A., M.I.E.E.

(RA-43)

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THE POLARIZATION OF WAVES REFLECTED FROM THE IONOSPHERE

SUMMARY

The effect of electron collisions on the polarization of waves reflected from the ionosphere is calculated by an iteration procedure and shown to be relatively small at night for both normal and oblique incidence, at frequencies above 0·5 MHz.

1. INTRODUCTION

The polarization of waves reflected from the ionosphere is of considerable importance at medium frequencies because only the vertically-polarized component of a downcoming wave couples efficiently with a receiving aerial. On the upward path the polarization which the ionosphere accepts is also of interest, since this may not be generated efficiently by the incident wave. The losses which arise between the incident wave and the ionosphere on the upward path, and between the downcoming wave and the receiving aerial, are called 'polarization coupling losses' and have been discussed elsewhere.[1] This report is concerned with the actual polarization of the wave accepted by the ionosphere, and with the polarization of the emerging wave. Discussion is confined to the polarization of the downcoming wave, since the polarization of the wave accepted by the ionosphere on the reciprocal path differs only in that the sense of rotation of the polarization ellipse is reversed. The polarization of the downcoming wave is called the *limiting polarization.*

According to the magneto-ionic theory, the polarization of characteristic waves in the ionosphere (i.e. ordinary and extraordinary waves) is modified by electron collisions, but only appreciably if the electron-collision frequency is greater than the wave frequency.[2] The polarization of a downcoming wave would therefore be expected to change as it passes through the lowest part of the ionosphere, because here the collision frequency is greatest. However Eckersley and Millington[3] have shown experimentally that the effect of collisions on the polarization of downcoming medium-frequency waves at night is relatively small; on the basis of this and other evidence collisions were neglected in the study of polarization coupling loss.[1] Booker[4] has shown theoretically that the limiting polarization of a downcoming wave is determined at the height where the refractive index gradient is comparable with $k|\mu_o - \mu_x|$, where μ_o and μ_x are the refractive indices of the ordinary and extraordinary waves respectively, $k = 2\pi/\lambda$ and λ is the wavelength in free space, and Booker's criterion was subsequently stated in a more precise form by Budden.[5] Bradley[6] assumed the limiting polarization of the wave to be the same as the characteristic polarization at the height given by Budden's criterion, and concluded that collisions have a significant effect on the polarization of downcoming waves both during the day and at night, especially at medium frequencies. This conclusion conflicts with the experimental evidence.

2. LIMITING POLARIZATION AT VERTICAL INCIDENCE

Consider a single characteristic wave (for example, a pure ordinary wave) descending into the part of the ionosphere where the polarization is modified by collisions. Here it is convenient to resolve the wave into two component waves (ordinary and extraordinary) each having the characteristic polarization for this region. The two waves may then be considered separately until they leave the ionosphere, when they may be recombined to give the wave whose polarization is observed. If the propagation constants of the two component waves do not differ greatly the observed polarization will be similar to that of the incident wave but if they do differ the polarization will be changed. This argument is a simplification because characterisitc polarization varies continually with height in the lower ionosphere, but if the ionosphere is regarded as a set of uniform horizontal layers the principle described may be applied to each layer in turn.

Let the z-axis of a rectangular co-ordinate system be parallel to the direction of propagation. The (complex) polarization of a wave R is then defined as $R = E_x/E_y$, where E_x and E_y are the electric fields in the plane of the wavefront. Consider a wave propagating vertically downwards through a set of layers and suppose that it encounters a layer (at height h) where the polarizations of the ordinary and extraordinary waves would be $R_o(h)$ and $1/R_o(h)$ if the medium were of infinite extent. At the upper boundary of the layer the incident wave may be resolved into ordinary and extraordinary waves distinguished by subscripts o and x respectively. The electric fields in the plane of the wavefront satisfy the following conditions:

$$E_x = E_{x\text{o}} + E_{x\text{x}} \tag{1}$$

$$E_y = E_{y\text{o}} + E_{y\text{x}} \tag{2}$$

$$E_{x\text{o}} = R_\text{o} E_{y\text{o}} \tag{3}$$

$$E_{y\text{x}} = R_\text{o} E_{x\text{x}} \tag{4}$$

From Equations (1) to (4) it may be shown that the component field of the two characteristic waves are related to the incident wave as follows:

$$E_{xo} = \frac{1-RR_o}{1-R_o^2} R_o E_y \qquad (5)$$

$$E_{yo} = \frac{1-RR_o}{1-R_o^2} E_y \qquad (6)$$

$$E_{xx} = \frac{R-R_o}{1-R_o^2} E_y \qquad (7)$$

$$E_{yx} = \frac{R-R_o}{1-R_o^2} R_o E_y \qquad (8)$$

If the complex refractive indices of the ordinary and extraordinary waves within the layer are n_o and n_x respectively, the propagation constants of the waves are $-jkn_o$ and $-jkn_x$. If the thickness of the layer is δh, the component fields at the bottom of the layer may be obtained by multiplying Equations (5) to (8) by $\exp(-jkn_o\delta h)$ or $\exp(-jkn_x\delta h)$ as appropriate. The sum of the ordinary and extraordinary waves at the bottom of the layer is therefore

$$E'_x = \frac{1}{1-R_o^2}\left[R_o(1-RR_o)e^{-jkn_o\delta h} + (R-R_o)e^{-jkn_x\delta h}\right]E_y \qquad (9)$$

$$E'_y = \frac{1}{1-R_o^2}\left[(1-RR_o)e^{-jkn_o\delta h} + R_o(R-R_o)e^{-jkn_x\delta h}\right]E_y \qquad (10)$$

The effective polarization R' of the sum of the two waves at the bottom of the layer is therefore

$$R' = \frac{E'_x}{E'_y} = \frac{R_o(1-RR_o)e^{-jk(n_o-n_x)\delta h} + R - R_o}{(1-RR_o)e^{-jk(n_o-n_x)\delta h} + R_o(R-R_o)} \qquad (11)$$

where $n_o - n_x$, the difference between the complex refractive indices of the ordinary and extraordinary waves, is given by Equation (2) of Bradley's paper.

Equation (11) has been used here as the basis of an iteration procedure* which computes the polarization of a wave step-by-step as it proceeds downwards through the lower ionosphere. The computation is started at a height where collisions have an insignificant effect and the incident wave is assumed initially to be a true characteristic wave, usually the ordinary. The polarization at the bottom of the first layer is calculated from Equation (11) and the resulting value is taken as the incident polarization for the next layer. This procedure is continued downwards through the ionosphere until no further change in the polarization of the resultant wave takes place.

Bradley's exponential model profiles were used for the computations, but the method can be applied to any profile. The ionosphere was divided into layers 1 km thick; a further computation showed that the use of 0·5 km layers made no significant difference to the results.

Table 1 shows the result obtained at 1 MHz from Bradley's daytime profile, when the angle between the ray direction and the Earth's magnetic field, θ, is 60° and the gyromagnetic frequency is 1·4 MHz. The polarization of the incident wave was assumed to be 0 + j0·400, the value the ordinary wave would have in the absence of collisions.

TABLE 1

Height km	R_o	R'
90	0·015 + j0·400	0·015 + j0·400
88	0·020 + j0·400	0·020 + j0·400
86	0·026 + j0·401	0·026 + j0·401
84	0·034 + j0·402	0·034 + j0·402
82	0·046 + j0·403	0·046 + j0·403
80	0·060 + j0·405	0·060 + j0·406
78	0·080 + j0·408	0·083 + j0·408
76	0·105 + j0·415	0·104 + j0·421
74	0·138 + j0·426	0·124 + j0·437
72	0·179 + j0·446	0·138 + j0·455
70	0·229 + j0·481	0·144 + j0·470
68	0·281 + j0·538	0·145 + j0·479
66	0·319 + j0·624	0·145 + j0·484
64	0·326 + j0·724	0·144 + j0·486
62	0·300 + j0·815	0·143 + j0·487
60	0·255 + j0·884	0·143 + j0·487
58	0·207 + j0·930	0·143 + j0·487

The second column shows the ordinary-wave polarization at stated heights while the third column gives the resultant polarization calculated by the iteration procedure. It will be seen that the latter follows the ordinary-wave polarization quite closely down to 78 km and shows little further change below 68 km. Thus 68 km may be regarded in this example as the height of limiting polarization, and it is of interest to note that Fig. 4(*a*) of Bradley's paper, calculated from Budden's criterion, gives this height as 65 km. Table 1 shows, however, that R and R_o differ appreciably at 68 km. The conclusion therefore is that, although the height of limiting polarization is given by Budden's criterion, the polarization of the downcoming wave may differ significantly from the characteristic polarization for this height.

In Fig. 1(*a*) the limiting polarization at vertical incidence for 1 MHz in daytime, calculated by the method described, is compared with values given by Bradley in Fig. 6 of his paper. It will be seen that the polarization calculated by the iteration procedure does not differ greatly from that which would arise in the absence of collisions.

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* A computer programme was written by Mr. R.W. Lee.

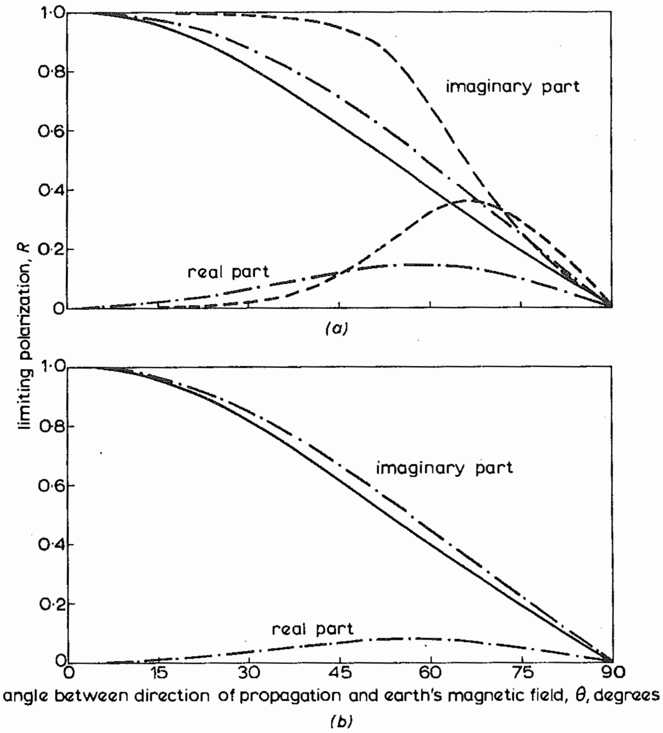

Fig. 1 - Limiting polarization at 1 MHz

(*a*) Normal incidence at daytime (*b*) Oblique incidence at night

———— Polarization, neglecting collisions (real part = 0)

—— • —— • Polarization calculated by iteration method

– – – – – – Polarization calculated by Bradley

Gyromagnetic frequency 1·4 MHz

3. LIMITING POLARIZATION AT OBLIQUE INCIDENCE

The iteration procedure may be extended to oblique incidence by replacing δh in Equation (11) by δh sec I, where I is the angle of incidence at the ionosphere. This procedure is justified provided the wave travels through the limiting polarization region of the ionosphere in an almost straight line, a condition satisfied at night for all practical angles of incidence at frequencies above 1 MHz. Under these circumstances Equations (1) to (4) still apply, and the modification to Equation (11) arises simply because the path length in each layer is increased by the factor sec I.

The effect of collisions on limiting polarization is greatest when sec I is large, but sec I cannot exceed 7 in the lower ionosphere because of Earth curvature; this value corresponds to waves arriving tangentially at the Earth's surface. Fig. 1(*b*), calculated by the iteration procedure for this extreme angle of incidence for 1 MHz at night, shows that the effect of collisions is again small. This indicates that the effect of collisions on polarization at night can always be disregarded at frequencies above 1 MHz. At lower medium frequencies (0·5 to 1 MHz), further calculations have shown that collision effects are unimportant at night except for waves arriving at angles very close to the horizontal.

4. COMPARISON OF THEORETICAL AND MEASURED POLARIZATIONS

In 1933 Eckersley and Millington measured the polarizations of waves received at night at Chelmsford, England from two medium-frequency transmitters situated in France.[3] A goniometer was used to measure the axial ratio of the polarization ellipse projected on the ground. Of the two transmissions, that from Paris on 914 kHz provided the more sensitive test of the effect of collisions on the polarization. Eckersley and Millington compared their measured results with the theoretical polarizations which would arise if collisions were entirely absent, or if they had the greatest possible effect, and concluded that the effect of collisions was small.

It is of interest to compare the measured value with that calculated by the iteration procedure, assuming Bradley's night-time electron-density profile and an angle of incidence at the ionosphere of 30°. Table 2 gives the result of this comparison, together with the values which would arise if collisions had no effect, or alternatively if they had the maximum possible effect.* Also included is the result obtained by Bradley's method, assuming the height at which limiting polarization occurs to be the same as that given by Bradley for vertical incidence.

TABLE 2

Axial Ratio of Polarization Ellipse Projected on the Ground

Theoretical, no collisions	0·348
Theoretical, calculated by the iteration procedure	0·359
Theoretical, calculated by Bradley's method	0·422
Theoretical, maximum collision effect	0·495
Measured	0·268 – 0·344

The theoretical axial ratios quoted in Table 2 are derived from the polarization of the ordinary wave, it being assumed that the extraordinary wave is heavily attenuated. The variation in the measured values may, however, be due to the presence of a weak extraordinary wave. It will be seen that the result calculated by the iteration procedure does not differ greatly from the result obtained if collisions are neglected, and lies closer to the measured values than does the result obtained from Bradley's method.

5. CONCLUSIONS

In calculating polarization coupling losses between the ionosphere and transmitting or receiving aerials it is customary to neglect the effect of electron collisions on the polarizations of waves entering or leaving the ionosphere.

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* The polarization of the downcoming wave would then be circular.

There is some experimental justification for this procedure but its validity has been questioned on theoretical grounds. The basis of the objection was that the polarization of a downcoming wave may be shown to be established at a height of about 70 km; at this height the polarizations of characteristic waves (ordinary and extraordinary waves) are significantly affected by collisions. It was thought that the polarization of a downcoming wave would be the same as the characteristic polarization at this height.

In the iteration procedure described here the polarization of a wave is calculated step-by-step as it proceeds downwards through the lower ionosphere. The results confirm that the polarization of the emerging wave is established at a height of about 70 km but they show that it is not the same as the characteristic polarization at that height. Moreover they show that the polarization of the emerging wave does not differ appreciably from that which would arise if collisions had no effect. This conclusion is valid at night for all practical angles of incidence at frequencies above 1 MHz; at frequencies between 0·5 and 1 MHz collision effects are significant only for waves arriving at angles very close to the horizontal. Consequently errors in polarization coupling loss calculations due to neglecting collisions are unlikely to be important at medium or high frequencies and the procedure which has been adopted in the past appears to be justified.

6. REFERENCES

1. PHILLIPS, G.J., *and* KNIGHT, P. 1964. Effects of polarization on a medium-frequency sky-wave service, including the case of multi-hop paths. *Proc.IEE*, 1965, **112**, pp. 31 – 39.
2. RATCLIFFE, J.A. 1959. The magneto-ionic theory and its applications to the ionosphere. · Cambridge University Press, 1959.
3. ECKERSLEY, T.L. *and* MILLINGTON, G. 1938. The limiting polarization of medium waves reflected from the ionosphere. *Proc. Phys. Soc.*, 1939, **51**, pp. 110 – 128.
4. BOOKER, H.G. 1936. Oblique propagation of electromagnetic waves in a slowly-varying non-isotropic medium. *Proc. Royal Soc.*, Series A, 1936, **155**, pp. 235 – 257.
5. BUDDEN, K.G. 1952. The theory of the limiting polarization of radio waves reflected from the ionosphere. *Proc. Royal Soc.*, Series A, 1952, **215**, pp. 215 – 233.
6. BRADLEY, P.A. 1967. Limiting polarization of radio waves of medium and high frequency reflected from the ionosphere. *Proc. IEE*, 1967, **114**, pp. 1837 – 1841.

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